647510 **BSCmaMJ208x-4C** **Seat No :________**

B.Sc. Semester - 4 (NEP-2020) Examination
March/April-2026 (As Per Syllabus Year June-2024)
MATH: Linear Algebra-2 (Major-8)

Time: 2:00 Hours **Marks: 50**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1 Answer any two out of three. (10)

(1) Verify that Linear Transformation on R^3, $T(a_1,a_2,a_3)=(a_1+3a_2+5a_3,\ 3a_1+2a_2+7a_3,\ 5a_1+7a_2+11a_3)$ is symmetric Linear Transformation or not.

(2) In usual notation prove that $R_T=(N_T)^{\perp}$, where T is symmetric Linear Transformation on inner product space V.

(3) Convert matrix $\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ -2 & 2 & 1 \end{bmatrix}$ in to diagonalize matrix.

Que.2 Answer any two out of three. (10)

(1) Find matrix for bilinear form $\varphi(x,y)= x_1y_1 +2x_1y_2 - 2x_2y_1 + x_2y_2$ with respect to base $\{(1,1), (1,-1)\}$.

(2) If φ_1 and φ_2 are bilinear forms on R^n then prove that $\varphi_1 + \varphi_2$ is also a bilinear form.

(3) If φ is a bilinear form then prove that there exist unique matrix A such that $\varphi(x,y)= xAy^T$.

Que.3 Answer any two out of three. (10)

(1) Find Reduction form of $Q(x)=x_1^2+ x_3^2+ 6x_2x_3 - 2x_3x_1 + 4x_1x_2$.

(2) Find rank, power and signature of $Q(x)= -2x_2x_3 + 2x_3x_1 + 2x_1x_2$.

(3) Prove that for symmetric matrices A_n and B_n there exist orthogonal matrix P_n such that $B=P^TAP$ if and only if characteristic values of A and B are same.

Que.4 Answer any two out of three. (10)

(1) Define full linear group. Prove that set of all orthogonal Linear Transformation from V to V forms a group under composition of functions. Where V is a real inner product space.

(2) Prove that Linear Transformation T on real inner product space V is orthogonal iff for all $u,v \in V$, $T(u) \cdot T(v)=u \cdot v$

(3) Check whether function $H: R^2 \dashrightarrow R^2$, $H(a, b)=(a+2b-3,\ 3a+b+5)$ is an affine transformation or not, if yes then find its inverse.

Que.5 Answer any two out of three. (10)

(1) Prove that, Characteristic value of a symmetric Linear Transformation is real.

(2) Verify that $\varphi(x,y)= x_1y_1 +2x_1y_2+ 3x_2y_1 +4x_2y_2$ is bilinear form in R^2 or not.

(3) If $A=\begin{bmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{bmatrix}$ is orthogonal then find values of l,m,n.

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